Micro III - June 2019 (Solution Guide)

1. Consider the following game G, where Player 1 chooses the row and Player 2 simultaneously chooses the column.

		Player 2		
		D	E	F
	A	$8,-1$	$2,0$	$1,0$
Player 1	B	$7,6$	$0,1$	$0,3$
	C	$2,2$	$4,3$	$0,0$

   (a) Show which strategies in G are eliminated by following the procedure of 'Iterated Elimination of Strictly Dominated Strategies'.

   SOLUTION: B is strictly dominated by A for Player 1, and can therefore be eliminated. After eliminating B, D is strictly dominated by E for Player 2, and can therefore be eliminated. No other strategy is strictly dominated for either player (although F is weakly dominated by E for Player 2).

   (b) Find all Nash equilibria (NE), pure and mixed, in G. Show which NE gives the highest payoff to *both* players, and denote this equilibrium strategy profile by $e(1)$.

   SOLUTION: There are two NE in pure strategies: (A, F) and (C, E). Moreover, there is also a NE where Player 1 plays A for sure; and Player 2 randomizes between playing E (with probability $q \leq 1/3$) and F (with probability $1-q$). The NE (C, E) gives the highest payoff to both players. Hence, $e(1) = (C, E)$.

   (c) Now consider the game $G(2)$, which consists of the stage game G repeated two times. For the time being, you can assume that players have discount factor $\delta = 1$, i.e. they place equal weight on period 1 and period 2 payoffs.

   Find one pure strategy Subgame Perfect Nash Equilibrium (SPNE) where *both* players are better off than they were in part (b). That is, where both players earn at least as much as in $e(1)$, in each period, and earn strictly more than in $e(1)$ in at least one period. (*NOTE: make sure to consider deviations in any subgame).* Denote the equilibrium strategy profile you found by $e(2)$.

   SOLUTION: Consider the following strategy s_1 for Player 1: 'Play B in period 1. Play C in period 2 if the period-1 outcomes was (B, D), and otherwise play A.' Consider the following strategy s_2 for Player 2: 'Play D in period 1. Play E in period 2 if the period-1 outcome was (B, D), and otherwise play F.' The strategy profile (s_1, s_2) implies NE play in period 2, in every subgame, so no deviation in period 2 can be profitable. To establish that (s_1, s_2) is a SPNE, it remains to show that no player has an incentive to deviate in period 1. Player 1 earns $7+4 = 11$ on the equilibrium path, whereas Player 2 earns $6+3 = 9$. Player 1's best deviation yields $8+1 = 9$, whereas Player 2's best deviation yields $3+0 = 3$. Hence, no deviation is profitable, which means (s_1, s_2) is a SPNE. Notice that, in period 2, both players earn the same payoff as in $e(1)$, and they both earn strictly more in period 1.

   (d) Now consider the game $G(\infty)$, which consists of the stage game G repeated infinitely many times. Assume that players discount future payoffs with factor $\delta \geq 1/7$.

Find one pure strategy SPNE where *both* players are better off than they were in part (c). That is, where both players earn at least as much as in $e(2)$, in each period, and earn strictly more than in $e(2)$ in at least one period.

SOLUTION: Consider the following strategy $Trigger_1$ *for Player* 1*: 'In period 1, play* B*. In any period* $t \geq 2$*, play* B *if* (B, D) *was the outcome of play in all periods* $t' < t$*; otherwise, play A.' Consider the following strategy* $Trigger_2$ *for Player* 2*: 'In period 1, play* D*. In any period* $t \geq 2$*, play* D *if* (B, D) *was the outcome of play in all periods* $t' < t$*; otherwise, play* F*.' The strategy profile* $(Trigger_1, Trigger_2)$ *implies NE play in every subgame after a deviation, so no deviation can be profitable off the equilibrium path. To rule out deviations on the equilibrium path, it is sufficient to consider both players' incentive to deviate in period 1. Player 1 earns* $7/(1-\delta)$ *on the equilibrium path, whereas Player 2 earns* $6/(1-\delta)$*. Player* 2*'s payoff from deviating in period 1 cannot exceed* $3 + 0\delta/(1-\delta) = 3$*, so such a deviation cannot be profitable. Player* 1*'s payoff from deviating in period 1 cannot exceed* $8 + \delta/(1-\delta)$*, and such deviation cannot be profitable when* $\delta \geq 1/7$*. Hence,* $(Trigger_1, Trigger_2)$ *is a SPNE. Notice that, in period 1, both players earn the same payoff as in* $e(2)$*, and they both earn strictly more in all later periods.*

2. Suppose we are in a private value auction setting. There are two bidders, $i = 1, 2$. They have valuation v_1 and v_2, respectively. These values are distributed independently uniformly with $v \sim U(3, 4)$. The auction format is sealed-bid first price. In case of a tie, a fair coin is flipped to determine the winner.

(a) Suppose player j uses the strategy $b(v_j) = cv_j + d$, where c and d are constants. Show that if bidder $i \neq j$ bids b_i, his probability of winning is

$$\mathbf{P}(i \ wins|b_i) = \frac{b_i - d - 3c}{c}$$

whenever $3c + d \leq b_i \leq 4c + d$.

Hint: Recall that if $x \sim U(a, b)$ then $\mathbf{P}(x \leq y) = \frac{y-a}{b-a}$

Solution: For $3c + d \leq b_i \leq 4c + d$*, we have* $P(i \ wins|b_i) = P(b_i > b(v_j)) = P(cv_j + d < b_i) = P(v_j < \frac{b_i - d}{c}) = \frac{b_i - d}{c} - 3 = \frac{b_i - d - 3c}{c}$.

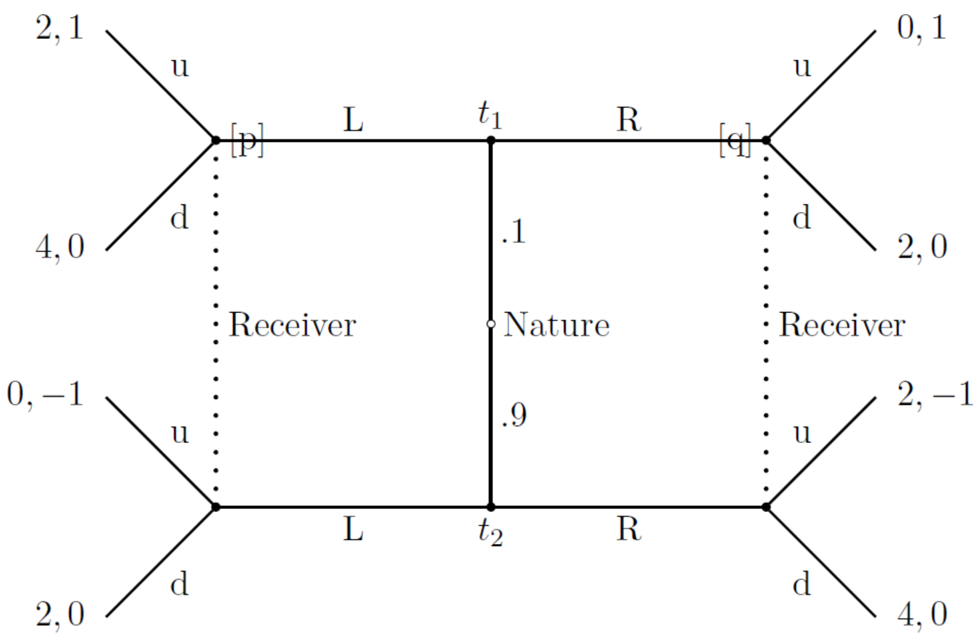
(b) Using the result in (a), show that there is a symmetric Bayes Nash equilibrium (BNE) in linear strategies $b(v_i) = cv_i + d, i = 1, 2$. Find c and d.

Solution: the expected payoff to bidder i from bidding b_i *is* $P(b_i > b(v_j))(v_i - b_i) = (\frac{b_i - d - 3c}{c})(v_i - b_i)$*. Taking the first-order-condition and solving for* b_i *yields* $b_i = \frac{v + d + 3c}{2}$*. Since we also know* $b_i = cv_i + d$*, matching coefficients gives* $c = \frac{1}{2}$*, and* $d = \frac{d + 3c}{2}$*, thus* $d = \frac{3}{2}$*. Conclusion: we have constructed a symmetric BNE in linear strategies,* $b(v_i) = \frac{v_i}{2} + \frac{3}{2}, i = 1, 2$.

(c) Check that, in the symmetric BNE from part (b), the lowest-type bidder will place a bid equal to his valuation, whereas all other bidders will bid strictly less than their valuation, and comment briefly on why this is the case. Does this relate to the idea of the 'winner's curse'? Briefly justify your answer (*NOTE: please attempt this sub-question even if you did not complete part (b)*).

Solution: notice that $b(v_i) = \frac{v_i}{2} + \frac{3}{2}$ *implies* $b(v_i = 3) = 3$ *and* $b(v_i) < v_i$ *for all* $v \in (3, 4]$*. A bidder with valuation* $v \in (3, 4]$ *will engage in bid-shading, by bidding* *less than her valuation. Bidding her valuation would give a payoff of zero, even if she wins the auction, whereas bidding lower can give her the possibility of winning the auction and earning a strictly positive payoff. In contrast, a bidder with valuation $v = 3$ is willing to bid her valuation; a higher bid clearly does not make sense, as it can possibly lead to losses, whereas a lower bid is not attractive because it would never allow her to win the auction. This idea of bid shading is unrelated to the winner's curse. The key point is that the winner's curse applies in a common value setting, where bidders should take into account that winning the auction constitutes 'bad news' about the value of the item in question; whereas here we look at a private value setting, where such concerns are not present.*

3. Now consider the following game G':



Note that in this game, the prior probability that the sender is of type 1 is equal to 0.1.

(a) Briefly explain whether G' is a static or a dynamic game (1 sentence), and whether or not G' is a cheap talk game (1 sentence).

SOLUTION: By definition, G is a dynamic game, where the Sender chooses a message and the Receiver then responds with an action. It is not a cheap talk game, because payoffs depend directly on messages.

(b) Find a separating equilibrium in G', and find a pooling equilibrium where both sender types play R.

SOLUTION: Separating equilibrium: $(LR, ud, p = 1, q = 0)$. *Pooling equilibria on* R*:* $(RR, ud, p \geq 1/2, q = 0.1)$.

(c) Check whether the two equilibria you found in part (a) satisfy Signaling Requirements 5 ('strict domination') and 6 ('equilibrium domination').

SOLUTION: The separating equilibrium satisfies both signaling requirements, since there are no off-the-equilibrium-path beliefs, and hence nothing to check. The pooling equilibrium satisfies Signaling Requirement 5, since no sender type has a strictly dominated strategy. The pooling equilibrium also satisfies Signaling Requirement 6 provided that off-equilibrium path beliefs are $p = 1$; t_2*'s equilibrium payoff of* 4 *is strictly higher than any payoff he could possibly get by deviating to* L, *whereas* t_1*'s equilibrium payoff of* 2 *is not. Signaling Requirement 6 therefore implies* $p = 1$, *which is consistent with* $p \geq 1/2$.

(d) Now suppose that we modify G', so that the prior probability of the sender being type t_1 is given by the parameter $\alpha \in [0, 1]$. For what values of α does a pooling equilibrium exist where both sender types play R? (*NOTE: parts (a) and (b) of this question considered the special case where* $\alpha = 0.1$).

SOLUTION: The pooling equilibrium in questions exists if and only if $\alpha \leq 1/2$.

(e) Briefly comment on why a pooling equilibrium where both sender types play R exists for some values of α, but not for others. What is the intuition for this result?

SOLUTION: The receiver would prefer to take action u *against a sender of type* t_1, *but to take action* d *against a sender type* t_2. *In a pooling equilibrium, on the equilibrium path, receiver beliefs are just given by the prior,* α. *Thus, the receiver is willing to play* d *in response to* R, *which is required in the pooling equilibrium from part (c), if* α *is sufficiently small, but will play* u *if* α *is sufficiently large. Intuitively, in the pooling equilibrium, a type 1 sender could be tempted to deviate to the message* L, *which gives him at least* 2, *whereas the message* R *gives him at most* 2. *To stop this deviation from being profitable, the sender needs to be rewarded after sending message* R. *The receiver can do so by playing* d. *But this will only occur in equilibrium if the receiver thinks it is sufficiently likely that the sender is type* t_2.